tangular duct computer program. However, as we approached these limits the configuration approached those of parallel infinite plates. In case 4, as AR tends to zero while B remains finite, Nu tends to infinity, because D_h tends to infinity.

In the triangular duct results shown in Fig. 3, the limiting cases are not as clearly indicated. For the case of constant q', Sparrow and Haji-Sheikh [2] indicated that the limiting Nu would be one-quarter of that obtained in the appropriate infinite plate solution. The results for our calculations, case 1, q' = constant, agreed closely with those presented by Sparrow and Haji-Sheikh [2] and are shown in Fig. 3.

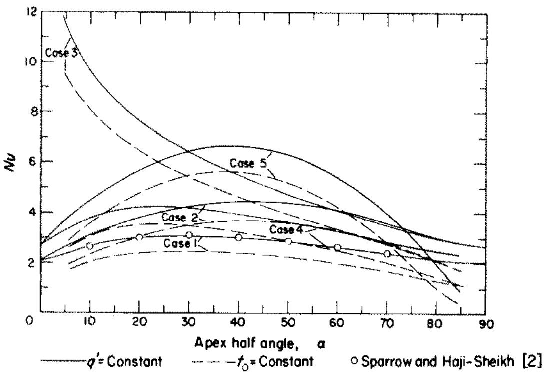

FIG. 3. Triangular duct.

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A METHOD OF CORRELATING LOCAL AND AVERAGE FRICTION COEFFICIENTS FOR BOTH LAMINAR AND TURBULENT FLOW OF GASES THROUGH A SMOOTH TUBE WITH SURFACE TO FLUID BULK TEMPERATURE RATIOS FROM 0·35 TO 7·35

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NOMENCLATURE

D, inside diameter of test section;
f, fanning friction coefficient;
L, test section length;
Re, Reynolds number;
T_b, bulk stagnation temperature;
T_f, film temperature $(T_s + T_b)/2$;
T_s, surface temperature;
X, distance from entrance of test section.

Subscript
s, denotes physical properties evaluated at the surface temperature.

INTRODUCTION

THERE has long been a need of a means of correlating both laminar and turbulent friction coefficients for gases with large variations in the physical properties flowing through smooth tubes. Probably the most widely used method of correlating and predicting friction coefficients for turbulent flow is that of Kármán–Nikuradse in which the friction coefficient and Reynolds number are evaluated at the film temperature T_f as postulated by Humble, Lowdermilk and Desmon [1]

$$\frac{1}{\sqrt{[8(f_f/2)]}} = 2\log\left(Re_f\Big/8\sqrt{\frac{f_f}{2}}\right) - 0{\cdot}8. \qquad (1)$$

This method worked well for Reynolds numbers greater than 20000, surface to fluid bulk temperature ratios (T_s/T_b) less than 2·5 and for a length-to-diameter ratio of 60. There are recent experiments which present friction coefficients which cannot be correlated by equation (1). In these experiments Taylor [2, 3] measured average friction coefficients for helium and hydrogen with wall temperatures up to 5600°R and T_s/T_b from 1·0 to 4·1, and Perkins and Worsøe-Schmidt [4] measured local friction coefficients for nitrogen with T_s/T_b from 1·4 to 7·35. It can be seen from Fig. 1 that the measured friction coefficient can be as much as three times the value predicted by equation (1). Perkins *et al.* found that some of their average friction data (T_s/T_b less than 2·46) could be correlated using T_s/T_b raised to the 0·6 power.

Dalle-Donne and Bowditch [5] showed that the bulk friction coefficient plotted as a function of the modified surface Reynolds number agreed well with both the laminar line and the Kármán–Nikuradse line for $1 < T_s/T_b \leqslant 2{\cdot}2$. When the method of correlation of [5] was extended to T_s/T_b as great as 7·35 the experimental data fell as much as 70 per cent below the Kármán–Nikuradse line as shown in Fig. 2.

Other experiments such as Davenport [6] presents heating data for helium and nitrogen in the laminar region. For turbulent flow, McEligot [7] and Magee [8] give heating data for helium and air, Wolf [9] gives both heating and cooling data for air and carbon dioxide, and Wolf and McCarthy [10] gives heating data for helium and hydrogen. The range of conditions covered by the various investigators is shown in Table 1.

This paper presents methods of correlating both local and average friction coefficients for laminar and turbulent flow of gases through smooth tubes with surface to fluid bulk temperature ratios from 0·35 to 7·35; and for the gases, helium, hydrogen, nitrogen, air and carbon dioxide, which represent the work of seven investigators.

The equations used in this investigation to correlate the friction data can very easily be used to predict friction coefficients for designers working within the range of test conditions shown in Table 1.

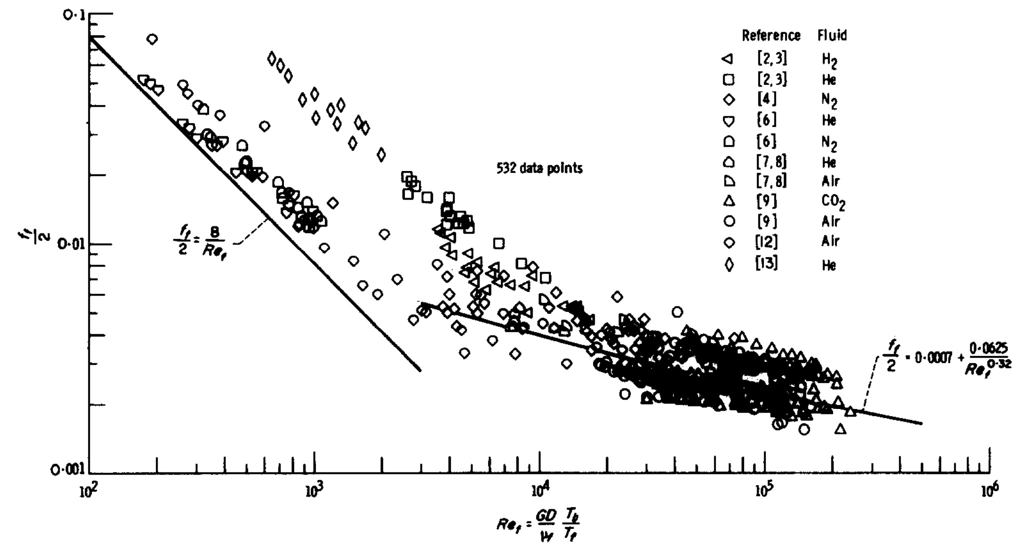

FIG. 1. Variation of local and average friction coefficients with modified film Reynolds number. Viscosity and density evaulated at the film temperature.

METHOD OF CORRELATION

Dalle-Donne and Bowditch were able to correlate friction coefficients for both laminar and turbulent flow of air and helium with small variations in the physical properties ($1 < T_s/T_b \leqslant 2{\cdot}2$) by using the bulk friction coefficient and the modified surface Reynolds number. In the present investigation this correlation method is used to check its validity for conditions where the variations in the physical properties are large due to either heating ($T_s/T_b > 1$) or cooling ($T_s/T_b < 1$) of the gas.

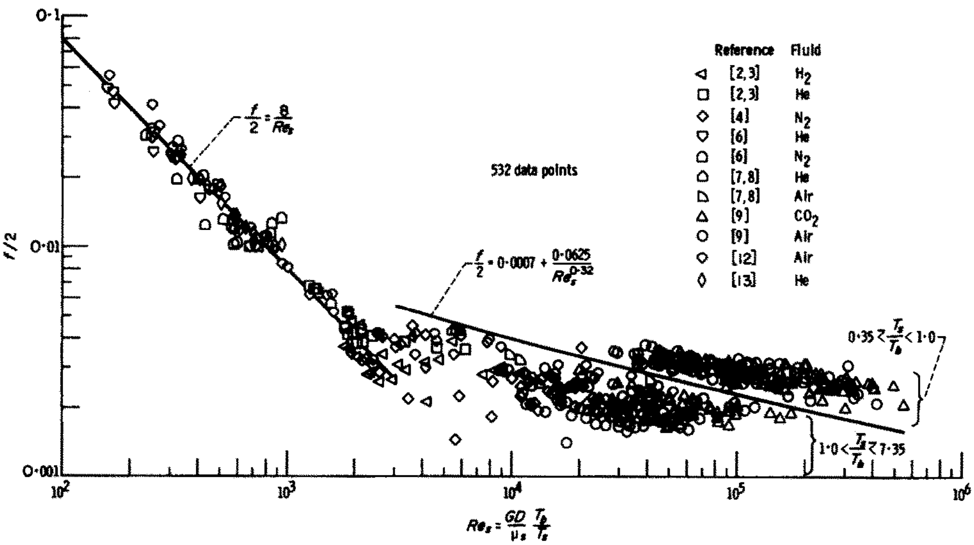

FIG. 2. Variation of local and average friction coefficients with modified surface Reynolds number. Density in friction coefficient evaluated at the bulk temperature. Viscosity in Reynolds number evaluated at the surface temperature.

Table 1. Test conditions for various sources of data

Ref.	Fluid	T_s/T_b	Type, $f/2$	L/D or X/D	T_b, (°R)	T_s, (°R)	$Re_s \times 10^{-3}$	Tube orientation
[2]	H_2 and He	1·0–3·7	Ave	80	829–1643	1559–4749	1·9–10·9	Vertical
[3]	H_2 and He	1·0–4·1	Ave	80	501–1243	1803–4600	1·3–4·2	Vertical
[4]	N_2	1·4–7·4	Loc	16, 53, 73, 93, 113	210–1091	371–2003	1·9–45·6	Vertical
[6]	N_2 and He	1·1–2·2	Loc	25, 44, 68	721–2226	1084–2460	0·17–0·94	Vertical
[7, 8]	He and Air	1·4–2·5	Loc	16, 18, 27, 34, 46, 63	541–1022	844–1705	5·1–40·9	Vertical
[9]	Air Co_2	0·35–2·7	Ave	21, 40, 60	545–1941	544–1782	11·3–550	Horizontal
[12]	Air	1·0–1·5	Ave	200	592–1324	711–1896	0·17–91·6	Horizontal
[13]	He	1·0–3·4	Ave	80	1045–1589	3112–5400	0·37–1·3	Vertical

The data of reference [10] is an order of magnitude lower than the data of all other investigators even at $T_s/T_b \approx 2{\cdot}3$ and are not used in this investigation.

LAMINAR FLOW

The relationship recommended by Dalle-Donne and

$$\frac{f}{2} = \frac{8}{Re_s}. \tag{2}$$

Equation (2) was used in this investigation to correlate friction data for helium, hydrogen, nitrogen and air with $1 < T_s/T_b \leqslant 4{\cdot}1$ with good results for modified surface Reynolds numbers up to 3000 (bulk Reynolds numbers up to 22000) as shown in Fig. 2.

TURBULENT FLOW

Dalle-Donne and Bowditch also found that the use of the bulk friction coefficient and the modified surface Reynolds number correlated their friction data for turbulent flow with $1 < T_s/T_b \leqslant 2{\cdot}2$ to the Kármán–Nikuradse line with good results. In the present investigation the Koo, Drew and McAdams relationship was used because of its simplicity and its close agreement with the Kármán–Nikuradse relationship.

The Koo, Drew and McAdams relationship [11] for bulk

Reynolds numbers from 3000 to 3000000 is given as

$$\frac{f}{2} = 0{\cdot}0007 + \frac{0{\cdot}0625}{Re_b^{0{\cdot}32}} \tag{3}$$

however, in the present investigation, the modified surface Reynolds number was used in place of the bulk Reynolds number giving the following relationship

$$\frac{f}{2} = 0{\cdot}0007 + \frac{0{\cdot}0625}{Re_s^{0{\cdot}32}}. \tag{4}$$

When equation (4) was used to predict friction coefficients for $0{\cdot}35 \leqslant T_s/T_b \leqslant 7{\cdot}35$ and modified surface Reynolds numbers from 3000 to 550000, the measured values were found to deviate from the predicted values as much as -70 per cent for the runs with heat addition ($1{\cdot}0 < T_s/T_b \leqslant 7{\cdot}35$) and $+70$ per cent for the runs with heat removal $0{\cdot}35 \leqslant T_s/T_b < 1{\cdot}0$ as shown in Fig. 2. Since the deviation appeared to be a function of T_s/T_b, the ratio of the friction coefficient calculated by equation (4) to the measured friction coefficient was plotted as a function of T_s/T_b and is shown in Fig. 3. The slope of the line drawn through the data points is 0·5 and passes through $(f/2)_{calc}/(f/2)_{exp} = 1$ at $T_s/T_b = 1$ as it should since equation (3) is equal to equation (4) at a $T_s/T_b = 1$. Figure 3 indicates that equation (4) should be modified with the square root of T_s/T_b as follows

$$\frac{f}{2} = \left(0{\cdot}0007 + \frac{0{\cdot}0625}{Re^{0{\cdot}32}}\right)\left(\frac{T_b}{T_s}\right)^{0{\cdot}5}. \tag{5}$$

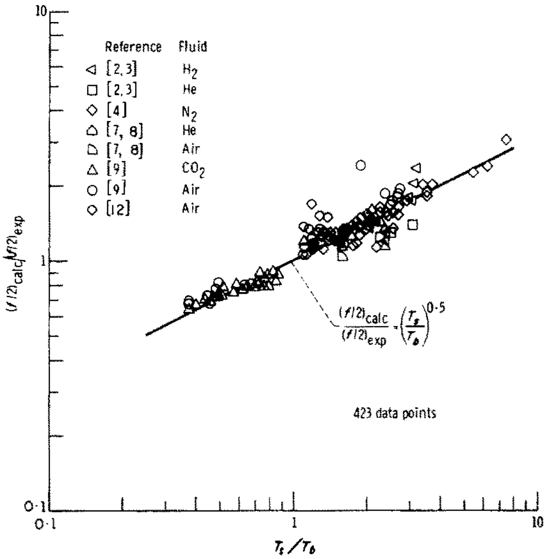

FIG. 3. Variation of the ratio of the friction coefficient calculated by equation (4) to the measured local and average friction coefficient with the ratio of surface to fluid bulk temperatures for modified surface Reynolds number of 3000 and greater.

Equation (5) predicts friction coefficients within ± 10 per cent for both heating and cooling of gases with modified surface Reynolds numbers from 3000 to 550000 (bulk Reynolds numbers from about 5400 to 187000, the limit of the experimental data).

DISCUSSION OF RESULTS

The relationships shown in equations (2) and (5) correlate local friction coefficients for X/D from 16 to 113, average friction coefficients for L/D from 21 to 200, for $0{\cdot}35 \leqslant T_s/T_b \leqslant 7{\cdot}35$ and modified surface Reynolds numbers from 170 to 550000 for both vertical and horizontal smooth tubes.

In Fig. 4, the experimental friction coefficients are shown as a function of the modified surface Reynolds number for low Reynolds number flow ($Re < 3000$). Of the 109 experimental points shown, 84 per cent of them fall within ± 20 per cent of the correlation line represented by equation (2).

Figure 5 shows the experimental friction coefficients as a function of the modified surface Reynolds number for turbulent flow (Re_s of 3000 and greater). There are 423

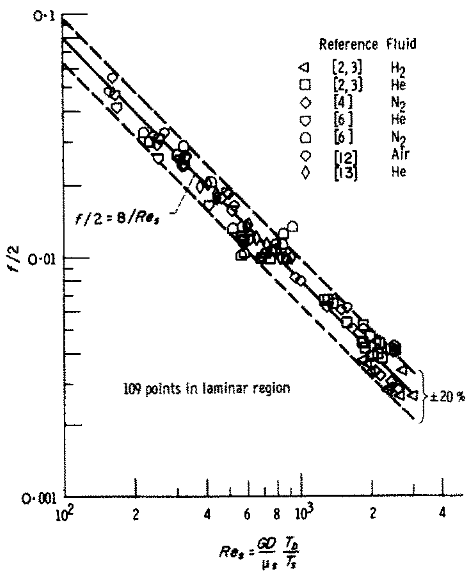

FIG. 4. Correlation of local and average friction coefficients for modified surface Reynolds number less than 3000. Density in friction coefficient evaluated at the bulk temperature. Viscosity in Reynolds number evaluated at the surface temperature.

experimental data points for both heating and cooling of the gas; 97 per cent of the data points are within ± 15 per cent and 90 per cent fall within ± 10 per cent of the correlation line defined by equation (5).

The data in Fig. 5 indicate a lack of the "transition region" except for the few points with a $T_s/T_b < 1{\cdot}3$ which fall below the correlation line. It appears that the large variations in phyical properties and density tends to make the flow turbulent at modified surface Reynolds numbers of 3000 or more.

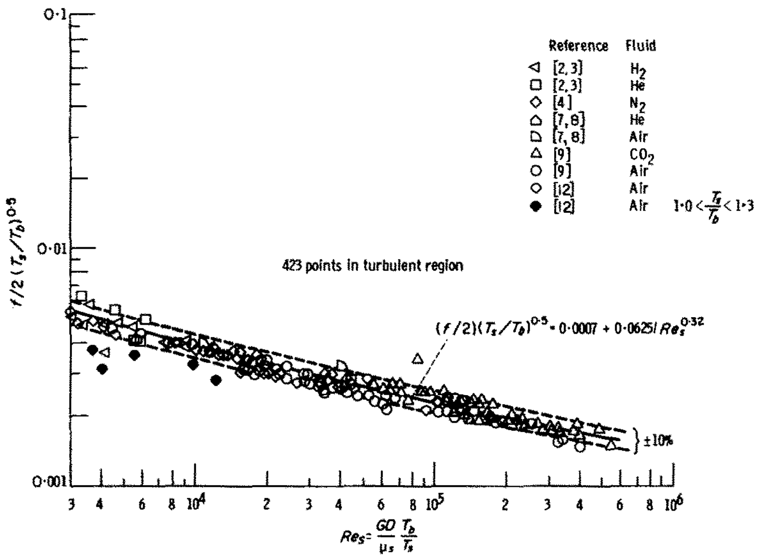

FIG. 5. Correlation of local and average friction coefficients for modified surface Reynolds number of 3000 and greater. Density in friction coefficients evaluated at the bulk temperature. Viscosity in Reynolds number evaluated at the surface temperature.

CONCLUSIONS

For the range of conditions shown in Table 1, the following conclusions may be made for flow through smooth circular tubes at X/D greater than 15.

1. The use of the reference temperature concept is not applicable to friction coefficients for flow through smooth tubes.

2. It is the modified surface Reynolds number which determines whether the laminar flow equation (2) or the turbulent flow equation (5) should be used to predict friction coefficients.

3. For modified surface Reynolds numbers below 3000, both local and average friction coefficients for $1{\cdot}0 < T_s/T_b \leqslant 4{\cdot}1$ can be predicted by the Dalle-Donne and Bowditch relationship $f/2 = 8/Re_s$.

4. For modified surface Reynolds numbers of 3000 and greater, the flow appears to be fully turbulent and both local and average friction coefficients for $0{\cdot}35 \leqslant T_s/T_b \leqslant 7{\cdot}35$ can be predicted by the relationship $f/2 = [0{\cdot}0007 + (0{\cdot}0625/Re_s^{0{\cdot}32})](T_b/T_s)^{0{\cdot}5}$.

5. For surface to fluid bulk temperature ratios greater than about 1·3 there appears to be no "transition region".

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